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THEORY OF A RADIAL GAS BEARING WITH CIRCULAR SUPPLY GROOVES

by

A. I. Snopov

FOREIGN TECHNOLOGY DIVISION

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13. ABSTRACT

It is known that a substantial increase in the load capacity of a self-generating gas bearing can be obtained at a corresponding increase in the gas pressure on the bearing ends only. This can be realized structurally by blowing the gas into the bearing trough through two annular grooves in the bushing, by locating the grooves near by the ends, and by maintaining a constant and equal gas pressure in them. In this case, the middle section of the bearing, inclosed between the grooves, will act as a self-generating bearing, and the end sections will operate under conditions of an axial pressure drop. A method of calculating the Reynolds equation is presented, permitting a solution of the problem to be obtained in relatively simple close approximations. An analytical solution of the equations of successive approximations can be plotted for the middle section of the bearing, but the pressure determination in the end sections generally requires the numerical inteygration of the ordinary differential equations. (AR0010701)

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III

THEORY OF A RADIAL GAS BEARING WITH
CIRCULAR SUPPLY GROOVES

[Article by A. I. Snopov; Moscow, Doklad na Soveshchanii po Gazovoy Smazke Podshipnikov, Russian, 12-14 February 1968, pp 63-70]

It is known that an essential increase in the load-bearing capacity of a self-generating gas bearing may be achieved only with a corresponding increase of gas pressure on the bearing face. Structurally, this may be accomplished if the gas inblow into the bearing is done through two circular grooves on the bushing, situated close to the end faces, and if the pressure of feeding gas is maintained stable and constant. In this case, the middle portion of the bearing between the grooves will work as a self-generating bearing, and the end segments will work under conditions of axial pressure drop.

Given is a method of solving Reynolds equation - p^2h^2 which determines pressure distribution in the lubricating layer. This method permits obtaining a relatively simple solution of the problem in high approximations. For the middle part of the bearing, an analytical solution of the equations of consecutive approximations may be structured, and the determination of pressures at the end sections requires, in a general case, numerical integration of the ordinary differential equations appearing at each iteration stage.

1. Formulation of the Problem and Basic Equations

Design computation of a radial bearing with several circular supply grooves requires the determination of pressure distribution both in the sections between grooves and at the end sections. We shall examine one such section. Let L be the section length, r_o the shaft radius, r_1 the bearing radius, ω the angular velocity of shaft rotation, and p_* and

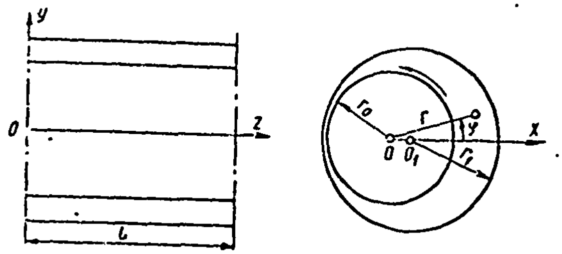

Figure 1.

p_{**} the pressures maintained at the section ends ($p_* \geq p_{**}$). We consider the gas to be isothermal, movement as stabilized, and the axes of the shaft and the bushing to be parallel. We shall make use of a Cartesian system of coordinates, with the z axis corresponding to the shaft axis and the x axis in the cross-section with pressure p_* and intersecting the bearing axis. Together with this, we shall use cylindrical coordinates in which Reynolds equations describing movement of the lubricant have the form

$$\mu = \frac{\partial^2 v_\varphi}{\partial r^2} = \frac{1}{r_0}\frac{\partial p'}{\partial \varphi}, \quad \mu\frac{\partial^2 v_z}{\partial r^2} = \frac{\partial p'}{\partial z'}, \quad \frac{\partial p'}{\partial r} = 0,$$
$$\frac{\partial(\rho' v_r)}{\partial r} + \frac{1}{r_0}\frac{\partial(\rho' v_\varphi)}{\partial \varphi} + \frac{\partial(\rho' v_z)}{\partial z'} = 0, \quad p' = c\rho'. \tag{1}$$

Solution of these equations in the investigated case should satisfy the conditions

$$v_r = v_z = 0, \quad v_\varphi = \omega r_0 \quad \text{at} \quad r = r_0,$$
$$v_r = v_z = v_\varphi = 0 \quad \text{at} \quad r = r_1 + e\cos\varphi; \tag{2}$$
$$p' = p_* \quad \text{at} \quad z' = 0, \quad p' = p_{**} \quad \text{at} \quad z' = L$$

(e -- eccentricity).

For convenience, we shall convert to nondimensional variables, which we shall introduce in the following manner:

$$\xi = \frac{r_1 - r_0}{\delta}, \quad \rho' = \rho_* \rho, \quad p' = p_* p, \quad p_* = c\rho_*, \quad z' = r_0 z,$$
$$L = r_0 \ell, \quad \delta = r_1 - r_0, \quad v_r = \omega\delta u_r, \quad v_\varphi = \omega r_0 u_\varphi, \quad v_z = \omega r_0 u_z, \tag{3}$$
$$\gamma = \frac{p_{**}}{p_*}, \quad A = \frac{p_*\delta^2}{\mu\omega r_0^2}, \quad b = \frac{e}{\delta}.$$

With this, equations and boundary conditions assume the form

$$\frac{\partial^2 u_\varphi}{\partial \xi^2} = \theta \frac{\partial p}{\partial \varphi}, \quad \frac{\partial^2 u_z}{\partial \xi^2} = \theta \frac{\partial p}{\partial z}, \quad \frac{\partial p}{\partial \xi} = 0,$$
$$-\frac{\partial (p u_r)}{\partial \xi} + \frac{\partial (p u_\varphi)}{\partial \varphi} + \frac{\partial (p u_z)}{\partial z} = 0, \tag{4}$$

$$u_r = u_z = 0, \quad u_\varphi = 1 \quad \text{at} \quad \xi = 1,$$
$$u_r = u_z = u_\varphi = 0 \quad \text{at} \quad \xi = -p\cos\varphi, \tag{5}$$
$$p = 1 \quad \text{at} \quad z = 0, \quad p = \gamma \quad \text{at} \quad z = \ell.$$

We shall introduce new variables

$$\zeta = (\xi + p\cos\varphi)h^{-1}, \quad q = ph, \quad h = 1 + p\cos\varphi. \tag{6}$$

With these variables, equations (4) and boundary conditions (5) are written thus

$$\frac{\partial^2 u_\varphi}{\partial \zeta^2} = \theta h \frac{\partial q}{\partial \varphi} + \theta q p \sin\varphi, \quad \frac{\partial^2 u_z}{\partial \zeta^2} = \theta h \frac{\partial q}{\partial z}, \quad \frac{\partial q}{\partial \zeta} = 0,$$
$$u_\varphi\left(h\frac{\partial q}{\partial \varphi} + q p \sin\varphi\right) + q h \frac{\partial u_\varphi}{\partial \varphi} - (1-\zeta)\frac{\partial u_\varphi}{\partial \zeta} q p \sin\varphi + h\frac{\partial (q u_z)}{\partial z} = q\frac{\partial u_r}{\partial \zeta}.$$

$$u_r = u_\varphi = u_z = 0 \quad \text{at} \quad \zeta = 0,$$
$$u_r = u_z = 0, \quad u_\varphi = 1 \quad \text{at} \quad \zeta = 1, \tag{8}$$
$$q = h \quad \text{at} \quad z = 0, \quad q = \gamma h \quad \text{at} \quad z = \ell.$$

From the first two equations (7) taking into account (8) we find that

$$u_\varphi = \zeta + \frac{1}{2}a(\zeta^2 - \zeta), \quad u_z = \frac{1}{2}b(\zeta^2 - \zeta),$$

where

$$a = \theta h \frac{\partial q}{\partial \varphi} + \theta q p \sin\varphi, \quad b = \theta h \frac{\partial q}{\partial z}. \tag{9}$$

We shall multiply the third equation (7) by $d\zeta$ and we shall integrate it for ζ within the limits from 0 to 1. After some simple computations taking into account (8) we obtain equation

$$b = \frac{\partial q}{\partial \varphi} - \frac{\partial (qa)}{\partial \varphi} - \frac{\partial (qb)}{\partial z} = 0, \tag{10}$$

which, upon substitution into it of the values of magnitudes a and b, we shall convert to the form

$$h\frac{\partial^2 q^2}{\partial z^2} + h\frac{\partial^2 q^2}{\partial \varphi^2} + \frac{\partial q^2}{\partial \varphi} p\sin\varphi + 2q^2 p\cos\varphi - 2\lambda\frac{\partial q}{\partial \varphi} = 0, \tag{11}$$

where $\lambda = \frac{6}{\theta} = \frac{6\nu\omega}{P_*}\left(\frac{r_0}{\delta}\right)^2.$

2. p^2h^2 - Method

The form of equation (11) naturally suggests accepting $q^2 = p^2h^2$ as the unknown magnitude and substituting it and the magnitude q in the form of series according to the degree of the relative eccentricity

$$q^2 = \sum_{n=0}^{\infty} s_n p^n, \quad q = \sum_{n=0}^{\infty} q_n p^n. \tag{12}$$

From the condition $\left(\sum_{n=0}^{\infty} q_n p^n\right)^2 = \sum_{n=0}^{\infty} s_n p^n$

we find that

$$q_0 = \sqrt{s_0}, \quad q_n = \frac{1}{2q_0}\left(s_n - \sum_{k=1}^{n-1} q_k q_{n-k}\right). \tag{13}$$

In agreement with (11), functions s_n are determined sequentially from equations

$$\frac{\partial^2 s_n}{\partial z^2} + \frac{\partial^2 s_n}{\partial \varphi^2} - \lambda\frac{\partial}{\partial \varphi}\left(\frac{s_n}{q_0}\right) = -\lambda\frac{\partial}{\partial \varphi}\left(\frac{1}{q_0}\sum_{k=1}^{n-1} q_k q_{n-k}\right) - \left(\frac{\partial^2 s_{n-1}}{\partial z^2} + \frac{\partial^2 s_{n-1}}{\partial \varphi^2}\right)\cos\varphi - \frac{\partial s_{n-1}}{\partial \varphi}\sin\varphi - 2s_{n-1}\cos\varphi, \tag{14}$$

where functions s_n should satisfy conditions

$$s_0 = 1,\; s_1 = 2\cos\varphi,\; s_2 = \frac{1}{2} + \frac{1}{2}\cos 2\varphi,\; s_n = 0\,(n \geq 3) \text{ at } z = 0,$$
$$s_0 = \gamma^2,\; s_1 = 2\gamma^2\cos\varphi,\; s_2 = \left[\frac{1}{2} + \frac{1}{2}\cos 2\varphi\right]\gamma^2,\; s_n = 0\,(n \geq 3) \text{ at } z = \ell. \tag{15}$$

Assuming that $s_o = s_o(z)$, we find easily

$$s_0 = 1 - \varepsilon\frac{z}{\ell},\quad q_0 = \left(1 - \varepsilon\frac{z}{\ell}\right)^{\frac{1}{2}},\quad \varepsilon = 1 - \gamma^2. \tag{16}$$

At n = 1, equation (14) assumes the form

$$\frac{\partial^2 s_1}{\partial z^2} + \frac{\partial^2 s_1}{\partial \varphi^2} - \frac{\lambda}{q_0}\frac{\partial s_1}{\partial \varphi} = -2 s_0 \cos\varphi. \tag{17}$$

We assume

$$s_1 = a_1(z)\sin\varphi + b_1(z)\cos\varphi. \tag{18}$$

On the basis of (15) and (17), we have

$$\begin{gathered} a_1'' - a_1 + \frac{\lambda}{q_0} b_1 = 0,\quad b_1'' - b_1 - \frac{\lambda}{q_0} a_1 = -2 s_0, \\ a_1(0) = a_1(\ell) = 0,\quad b_1(0) = 2,\quad b_1(\ell) = 2\gamma^2. \end{gathered} \tag{19}$$

We shall introduce a complex function

$$w_1 = a_1 + i b_1 - 2 q_0^2 i,\quad i = \sqrt{-1}. \tag{20}$$

This function, in agreement with (19) satisfies conditions

$$w_1'' - w_1 - \frac{\lambda i}{q_0} w_1 = -2 i q_0,\quad w_1(0) = w_1(\ell) = 0. \tag{21}$$

Assuming that in (14) n = 2, we obtain

$$\begin{gathered} \frac{\partial^2 s_2}{\partial z^2} + \frac{\partial^2 s_2}{\partial \varphi^2} - \frac{\lambda}{q_0}\frac{\partial s_2}{\partial \varphi} = -\frac{1}{2} b_1'' + \left[-\frac{\lambda}{4 q_0^3}\left(a_1^2 - b_1^2\right) - \right. \\ \left. -\frac{1}{2} a_1'' - a_1\right]\sin 2\varphi + \left[-\frac{\lambda}{2 q_0^3} a_1 b_1 - \frac{1}{2} b_1'' - b_1\right]\cos 2\varphi. \end{gathered} \tag{22}$$

If it is assumed

$$s_2 = a_2(x)\sin 2\varphi + b_2(x)\cos 2\varphi + c_2 \quad , \tag{23}$$

then

$$
\begin{aligned}
& a_2'' - 4a_2 + \frac{2\lambda}{q_0} b_2 = -\frac{\lambda}{4q_0^3}\left(a_1^2 - b_1^2\right) - \frac{1}{2}a_1'' - a_1 \; , \\
& b_2'' - 4b_2 - \frac{2\lambda}{q_0} a_2 = -\frac{\lambda}{2q_0^3} a_1 b_1 - \frac{1}{2} b_1'' - b_1 \; , \\
& c_2 = -\frac{1}{2} b_1'' \; , \\
& a_2(0) = a_2(\ell) = 0, \; b_2(0) = \frac{1}{2}, \; b_2(\ell) = \frac{1}{2}\gamma^2, \; c_2(0) = \frac{1}{2}, \; c_2(\ell) = \frac{1}{2}\gamma^2 .
\end{aligned}
\tag{24}
$$

We shall introduce the function

$$w_2 = a_2 + ib_2 - \frac{1}{2} q_0^2 i \quad , \tag{25}$$

evidently, it is determined in relation to (24) from conditions:

$$w_2'' - 4w_2 - \frac{2\lambda i}{q_0} w_2 = -\frac{\lambda}{4q_0^3} w_1^2 - \frac{3}{2}\frac{\lambda i}{q_0} w_1 - \frac{3}{2} w_1 + \lambda q_0 , \quad w_2(0) = w_2(\ell) = 0. \tag{26}$$

We find easily also that

$$c_2 = -\frac{1}{2} b_1'' + \frac{3}{2} q_0^2 \quad . \tag{27}$$

We shall limit ourselves by the indicated two approximate values. Knowing w_1 and w_2 we may determine q_1 and q_2. As the result we find that

$$
\begin{aligned}
& q_1 = \frac{1}{2q_0} s_1 = A_1 \sin\varphi + B_1 \cos\varphi \; , \\
& q_2 = \frac{1}{2q_0} s_2 - \frac{1}{2q_0^3} s_1^2 = A_2 \sin 2\varphi + B_2 \cos 2\varphi + C_2 \; ,
\end{aligned}
\tag{28}
$$

where

$$A_1 + iB_1 = \frac{1}{2q_0} w_1 + q_0 i \; , \quad A_2 + iB_2 = \text{[illegible]} \; ,$$

$$C_2 = \frac{i}{4q_0}\left(w_1 - \bar{w}_1\right) - \frac{1}{16q_0^3} w_1 \bar{w}_1 \; , \quad \bar{w}_1 = a_1 - ib_1 + 2q_0^2 i \; .$$

In the general case when $\gamma \neq 1$, it will not be possible to obtain an analytical solution of equations (21) and (26) in a closed form, and for practical application one may take advantage of their numerical integration. In the case $\gamma = 1$, the solution may be presented in the hyperbolic functions of a complex argument [2], at which point we shall stop. We should note that in its first approximation it is in agreement with Osman's solution [1].

3. Lubrication Effect on the Shaft and Gas Consumption

The main vector of forces of pressure applied to the examined section of shaft has the following components

$$P_x = -r_0^2 p_* \int_0^l dz \int_0^{2\pi} \frac{q\cos\varphi}{h} d\varphi, \quad P_y = -r_0^2 p_* \int_0^l dz \int_0^{2\pi} \frac{q\sin\varphi}{h} d\varphi \qquad (29)$$

and we shall represent it in the following complex form

$$\begin{aligned} P_y + i\sqrt{1-p^2} P_x &= \frac{2\pi r_0^2 p_* (1-\sqrt{1-p^2})}{p} \int_0^l \Big[i(q_0 + p\, C_2) - \\ &- (A_1 + iB_1) + (1-\sqrt{1-p^2})(A_2 + iB_2) + \ldots \Big] dz = \\ &= \frac{2\pi r_0^2 p_* (1-\sqrt{1-p^2})}{p} \int_0^l \Big\{ -\Big[\frac{1}{4q_0}(w_1 - \bar{w}_1) + \frac{i}{16 q_0^3} w_1 \bar{w}_1\Big] - \\ &- \frac{1}{2q_0} w_1 + (1-\sqrt{1-p^2})\Big(\frac{1}{2q_0} w_2 + \frac{i}{16 q_0^3} w_1^2 - \frac{1}{4q_0} w_1\Big) + \ldots \Big\} dz. \end{aligned} \qquad (30)$$

The moment of the forces of friction applied to the shaft along section L, with respect to the z axis may be, as usual, represented by a formula

$$M = -\frac{2\pi r_0^3 L \mu \omega}{\delta\sqrt{1-p^2}} + \frac{e}{2} P_y. \qquad (31)$$

Computing the consumption of lubricant per second in the examined section from the formula

$$Q = \int_0^{2\pi}\int_{r_0}^{r_1 + z\cos\varphi} \rho v_z r\,dr\,d\varphi ,$$

we find that it is independent of the angular velocity of shaft rotation and equals

$$Q = \frac{\pi r_0 \delta^3 (P_*^2 - P_{**}^2)}{3\mu L} \frac{\rho_*}{P_*}\left(1 + \frac{3}{2}b^2 + \ldots\right). \tag{33}$$

Since at $\lambda = 0$, $w_1 = w_2 = 0$, then, in agreement with (30) and $P_x = P_y = 0$, consequently, in a bearing with a circular inblow, load-bearing capacity is produced by the shaft rotation.

4. Case of High Velocities of Shaft Rotation

In the case of high velocities of shaft rotation, λ is large and it is possible to construct an asymptotic solution of equations (21) and (26). It will contain the functions which are solutions of equations at $\lambda = \infty$ and boundary layer functions of the form $e^{-\sqrt{\lambda}kz}$, $e^{-\sqrt{\lambda}k(l-z)}$. When computing the integral (30), in the first approximation one may neglect the squares of boundary layer functions and retain only the integrals of threshold functions corresponding to $\lambda = \infty$.

Assuming in (21) and (26) $\lambda = \infty$, we easily find that the threshold solutions have the form

$$w_1 = -2iq_0^2 , \quad w_2 = -\frac{i}{2}q_0^2 . \tag{34}$$

At the same time

$$P_y + i\sqrt{1-b^2}\,P_x = \frac{2\pi r_0 L p_*(1-\sqrt{1-b^2})}{b}\left(1 + \frac{3}{4}b^2 + \ldots\right)\frac{1}{l}\int_0^l q_0\,dz. \tag{35}$$

But

$$\frac{1}{l}\int_0^l q_0\,dz = \frac{2}{3\varepsilon}\left[1-(1-\varepsilon)^{3/2}\right],$$

consequently, in the case of high velocity of the shaft rotation

$$P_x = \frac{2\pi r_0 L p_*(1-\sqrt{1-p^2})(1+\frac{3}{4}p^2+\dots)}{p\sqrt{1-p^2}} K(\varepsilon), \quad P_y = 0,$$
$$M = -\frac{2\pi r_0^3 L\mu\omega}{\delta\sqrt{1-p^2}}, \tag{36}$$

where $K(\varepsilon) = \frac{2}{3\varepsilon}\left[1-(1-\varepsilon)^{3/2}\right], \quad \varepsilon = 1-\left(\frac{p_{**}}{p_*}\right)^2$

while $K(0) = 1, \quad K(1) = \frac{2}{3}$.

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